Ultrasonic waves Reflected at Oblique Incidence by Porous Rigid Medium.

Mustapha Sadouki, Mohamed Fellah, Z.E.A Fellah, Claude L. Depollier

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M. Sadouki[a] , M. Fellah[c] , Z.E.A. Fellah[b] and C.Depollier[d]

a. Fac. des Sciences et de la Technologie, Université Djilali Bounaama à Khemis Miliana, BP 44225, Ain Defla, Algérie
b. LMA UPR7051 CNRS Aix-Marseille Univ, Centrale Marseille, 31 chemin Joseph Aiguier, F-13402 Marseille Cedex, 20, France
c. Lab. de Physique Théorique, Faculté de Physique, USTHB, BP 32, 16111 El Alia, Bab Ezzouar, Algérie.
d. LUNAM Université du Maine, UMR CNRS 6613 Laboratoire d'Acoustique de l'Université du Maine UFR.STS, Avenue O. Messiaen, 72085 Le Mans CEDEX 09, France / National Research University "MPEI" Moscow, Russia.

Abstract:

An ultrasonic reflected wave at oblique incidence by porous medium with rigid frames is considered using equivalent fluid model. The viscous and thermal losses of the medium are described by two susceptibility kernels which depend on the viscous and thermal characteristic lengths. Analytical derivation of reflection coefficient is given in frequency domain. The simulated reflected wave is obtained at time domain by convolution between the reflected operator and the incident field. Experimental results for plastic foam samples of air-saturated porous media are given and compared with theoretical prediction.

Keywords: Ultrasonic, Reflected wave, Oblique incidence, Porous medium, Rigid frame.

1 Introduction

Porous materials are ubiquitous in our environment; naturally, where soils and rocks are examples the most common; in industry, where the construction materials such as concrete and road surfaces are often used to reduce noise; in medicine, where the analysis of some living tissue such as the lungs or bone, requires description content of porous materials. Therefore in terms of applications, the materials porous represent considerable interest. That it is for analyzing seismic signals obtained in the oil industry or to optimize the efficiency an acoustic material, or to diagnose some diseases, the characterization of these materials is paramount. The determination of the properties of a medium from waves that have been reflected by or transmitted through the medium is a classical inverse scattering problem. The important parameters which appear in theories of sound propagation in porous materials [5,16] at high frequencies range are the porosity, the tortuosity[15], the viscous [2] and thermal [15] characteristic lengths. Porosity is the relative fraction, by volume, of air contained in the connected pores in the material and presents a key parameter playing an important role in the propagation at all frequencies. As such, in studies of acoustical properties of porous materials, it is highly desirable to be able to measure this parameter. Beranek [13] described an apparatus (porosimeter) used to measure the porosity of porous materials. This device was based on the equation of state for ideal gases at constant temperature (i.e., Boyle's law). Porosity can be determined by measuring the change in air pressure occurring with a known change in volume of the chamber containing the sample. The tortuosity, namely the structure factor *ks* by Zwikker and Kosten[14] or the parameter q[6] by Attenborough[12] is an important parameter which intervenes in the description of the inertial interaction between the fluid and the structure in the porous material at high frequency range. In the case of cylindrical pores making an angle θ with the direction of propagation $\alpha_\infty = 1/cos^2\theta$. The tortuosity can be evaluated by electrical measurements [17] or by using a superfluid ^{4}He as the pore fluid[8]. It can also be evaluated by acoustical techniques as an ultrasonics measurement of transmitted waves[6,7,17]. The viscous characteristic length Λ is a geometrical parameter introduce by Johnson et al on the characterization of viscous effects at high frequencies given by $\frac{2}{\Lambda} = \frac{\int_s u^2 ds}{\int_v u^2 dv}$. where u is the speed of a microscopic incompressible perfect of fluid. The definition of this parameter applies to a smooth interface fluid/solid, and for a low boundary layer thickness to the radius of curvature characteristic of the interface. When the pore surface has singularities (peaks), this definition of the characteristic length is no longer valid. The parameter Λ is an indicator of the size of the narrow neck of the pore, i.e. the privileged place of viscous exchanges. Allard and Champoux[1], introduced by analogy with Johnson *et al* [2], a geometric parameter called thermal characteristic length given by $\frac{2}{\Lambda'} = \frac{\int_s ds}{\int_v dv}$. The length Λ' is an indicator of the size of large pores, privileged place of heat exchange. The characteristic lengths can be deduced from the high-frequency asymptotic behavior of either the velocity or the attenuation curves obtained in the sample saturated by air and by helium [19].

In this work, we present temporal model for the propagation of ultrasonic reflected wave at oblique incidence in homogeneous isotropic slab of porous material with rigid frame. Analytical expression of reflection coefficient at oblique incidence is calculated at frequency domain, this expression depend on the porosity, tortuosity, viscous and thermal characteristic length as well as the incidence angle. Expression of reflection kernel in the time domain at oblique incidence is calculated. Finally, an experimental validation using ultrasonic measurement is performed for air-saturated plastic foam and compared with theoretical prediction.

2 Model

In porous material acoustics, a distinction can be made between two situations depending on where the frame is moving or not. In the first case, the wave dynamics due to coupling between the solid frame and fluid are clearly described by the Biot theory [10, 11]. In air-saturated porous media, the structure is generally motionless and the waves propagate only in the fluid. This case is described by the equivalent fluid model which is a particular case in the Biot model, in which fluid-structure interactions are taken into account in two frequency response factors: dynamic tortuosity of the medium α(ω) given by Johnson *et al.*[2] and dynamic compressibility of the air in the porous material

$\beta(\omega)$ given by Allard[1]. In the frequency domain, these factors multiply the fluid density and compressibility, respectively, and show the deviation from fluid behavior in free space as frequency increases. The ultrasonic regime corresponds to the range of frequencies such that viscous skin thickness $\delta = \sqrt{2\eta/\omega\rho_f}$ is much small than the radius of the pores r, $\frac{\delta}{r} \ll 1$. This is called also the high-frequency range. In the asymptotic domain (high frequency approximation) the expressions of the responses factors $\alpha(\omega)$ and $\beta(\omega)$ are given by[1, 2]:

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{2}{\Lambda}\left(\frac{\eta}{j\omega\rho_f}\right)^{\frac{1}{2}}\right), \tag{1}$$

$$\beta(\omega) = 1 + \frac{2(\gamma-1)}{\Lambda'}\left(\frac{\eta}{P_r\rho_f}\right)^{1/2}\left(\frac{1}{j\omega}\right)^{1/2} \tag{2}$$

In these equations, ω is the pulsation, Pr is the Prandtl number, η and ρ_f are, respectively, the fluid viscosity and the fluid density, and γ is the adiabatic constant. The relevant physical parameters of the model are the tortuosity of the medium α_∞ initially introduced by Zwikker and Kosten[14], the viscous and the thermal characteristic lengths Λ and Λ' introduced by Johnson *et al.*[2] and Allard[1].

3 Direct problem

The direct scattering problem is that of determining the scattered field as well as the internal field that arises when a known incident field impinges on the porous material with known physical properties. The reflected and transmitted fields are deduced from the internal field and the boundary conditions. The geometry of the problem is shown in Fig. 1.

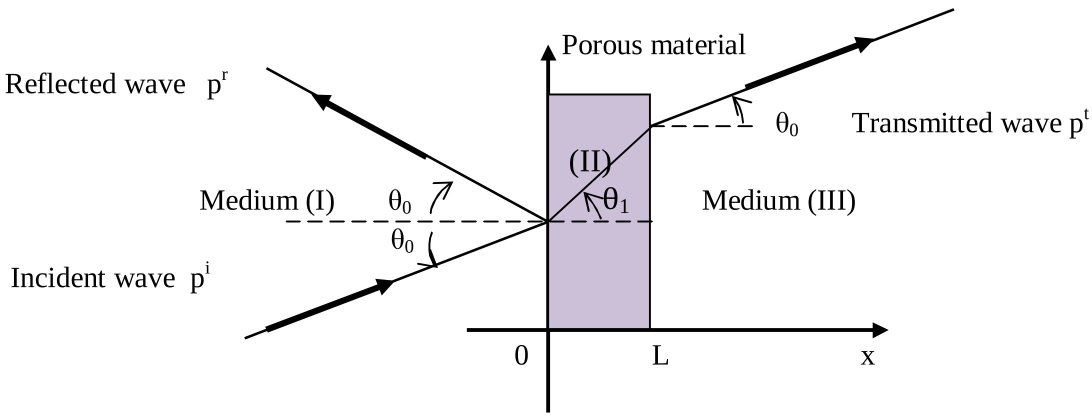


Figure 1 – Geometry of the problem.

A homogeneous porous material occupies the region $0 < x < L$. This medium is assumed to be isotropic and to have a rigid frame. A short sound pulse impinges at oblique incidence on the medium from the left, it gives rise to an acoustic pressure field $p(x,z,t)$ and an acoustic velocity field $v(x,z,t)$ within the material, which satisfying the Euler equation and the constitutive equation (along the x axis):

$$\rho\alpha(\omega)j\omega v = \nabla p, \qquad \frac{\beta(\omega)}{K_a}j\omega p = \nabla . v \tag{3}$$

where $j^2 = -1$, ρ is the suturing fluid density, and K_a is the compressibility modulus of the fluid. The expression of a pressure wave incident plane, unit amplitude, arriving at oblique incidence to the porous material is given by

$$p^i(x,z,\omega) = e^{-j(kxcos\theta_0+kzsin\theta_0)}e^{-j(-\omega t)}, \qquad (4)$$

where θ_0 is the incident angle, $k = \frac{\omega}{c_0} = \omega\sqrt{\frac{\rho}{K_a}}$, k is the wave number of the free fluid.

In the medium (I) (x < 0), the movement results from the superposition of incident and reflected waves:

$$p_1(x,z,\omega) = \left(e^{-jkxcos\theta_0} + R(\omega)e^{jkxcos\theta_0}\right)e^{-j(kzsin\theta_0-\omega t)} \qquad (5)$$

where $R(\omega)$ is the reflection coefficient.

According to Eq. (3), the expression of the velocity field in the medium (I) wrote:

$$v_1(x,z,\omega) = \frac{cos\theta_0}{Z_f}\left(e^{-jkxcos\theta_0} - R(\omega)e^{jkxcos\theta_0}\right)e^{-j(kzsin\theta_0-\omega t)} \qquad (6)$$

Where $Z_f = \sqrt{\rho K_a}$

In the medium (II) corresponding to the porous material, the expressions of the pressure and velocity field are:

$$p_2(x,z,\omega) = \left(A(\omega)e^{-jk_m xcos\theta_1} + B(\omega)e^{jk_m xcos\theta_1}\right)e^{-j(k_m zsin\theta_1-\omega t)} \qquad (7)$$

$$v_2(x,z,\omega) = \frac{cos\theta_1}{Z_m}\left(A(\omega)e^{-jk_m xcos\theta_1} - B(\omega)e^{jk_m xcos\theta_1}\right)e^{-j(k_m zsin\theta_1-\omega t)} \qquad (8)$$

In these expressions θ_1 is the refracted angle in the medium (II), $A(\omega)$ and $B(\omega)$ are functions of pulsation for determining, Z(ω) and k(ω) are the characteristic impedance and the wave number, respectively, of the acoustic wave in the porous medium. These are two complex quantities:

$$k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}}, \qquad Z(\omega) = \sqrt{\frac{\rho K_a\alpha(\omega)}{\beta(\omega)}} \qquad (9)$$

Finally, in the medium (III), the expression of the pressure and velocity fields of the wave transmitted through the porous material are

$$p_3(x,z,\omega) = T(\omega)e^{-jk(x-L)cos\theta_0}e^{-j(kzsin\theta_0-\omega t)}, \qquad (10)$$

$$v_3(x,z,\omega) = \frac{cos\theta_0}{Z_f}T(\omega)e^{-jk(x-L)cos\theta_0}e^{-j(kzsin\theta_0-\omega t)} \qquad (11)$$

where $T(\omega)$ is the transmission coefficient.

To derive the reflection scattering operator, it is assumed that the pressure field and flow velocity are continuous at the material boundary:

$$p_1(0^-,\omega) = p_2(0^+,\omega) \qquad (12)$$

$$p_2(L^-,\omega) = p_3(L^+,\omega) \qquad (13)$$

$$v_1(0^-,\omega) = \phi v_2(0^+,\omega) \qquad (14)$$

$$\phi v_2(L^-,\omega) = v_3(L^+,\omega) \qquad (15)$$

where ϕ is the porosity of the medium and the ± superscript denotes the limit from right and left, respectively. Using boundary and initial condition (12)-(15), reflection coefficient can be derived:

$$R(\omega) = \frac{(1-E^2(\omega))sinh(jk(\omega)L)}{2E(\omega)cosh(jk(\omega)L)+(1+E^2(\omega))sinh(jk(\omega)L)} \tag{16}$$

where,

$$E(\omega) = \phi\frac{cos\theta_1}{cos\theta_0}\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}\ ,\ k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}}cos\theta_1 \text{ and } cos\theta_1 = \sqrt{1-\frac{sin\theta_0{}^2}{\alpha(\omega)\beta(\omega)}} \tag{17}$$

The incident and scattered fields are related by the scattering operators (i.e., the reflection operators) for the material. These are integral operators represented by:

$$\begin{aligned} p^r(x,t) &= \int_0^t \tilde{R}(\tau)p^i\left(t-\tau+\frac{x}{c_0}\right)d\tau. \\ &= \tilde{R}(t) * p^i(t) * \delta\left(t+\frac{xcos\theta_0}{c_0}\right) \end{aligned} \tag{18}$$

In Eq. (18) * denotes the convolution operation, the function $\tilde{R}(t)$ is the reflection kernel, for incidence from the left, its temporal expression is obtained numerically by taking the inverse Fourier transform of Eq.(16), $p^i(t)$ is the incident field and δ is the Delta function. Note that the lower limit of integration in Eq. (18) is chosen to be 0, which is equivalent to assume that the incident wave front first impinges on the material at t=0. The scattering operators given in Eq. (18) is independent of the incident field used in the scattering experiment and depends only on the properties of the materials.

4 Experimental validation

As an application of this model, some numerical simulations are compared to experimental results. Experiments are carried out in air with two broadband Ultran NCT202 transducers having a 190 kHz central frequency in air and a bandwidth at 6 dB extending from 150 kHz to 230 kHz. A goniometer used in optic has been employed for the positioning of the transducers. Pulses of 400 V are provided by a 5052PR Panametrics pulser/receiver. The received signals are amplified up to 90 dB and filtered above 1 MHz to avoid high frequency noise. Electronic perturbations are removed by 1000 acquisition averages. The experimental setup is shown in Fig. 2.

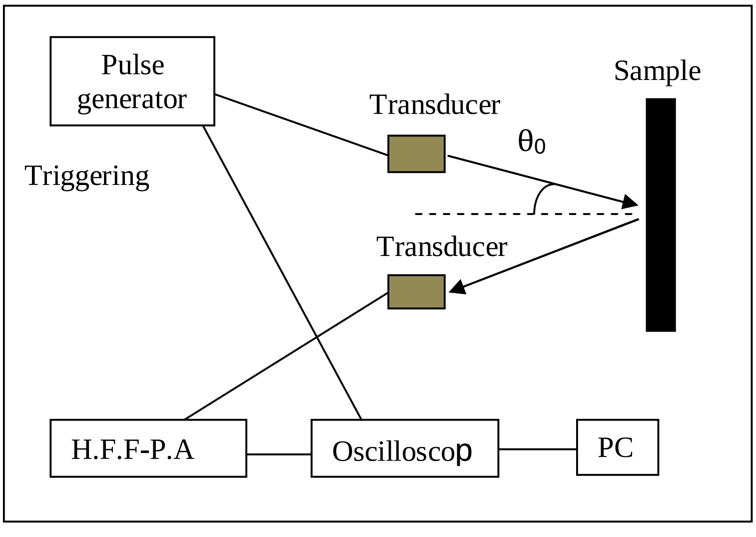


Fig.2. Experimental set-up of the ultrasonic measurement in reflected mode (H.F.F-P.A: high frequency filtering-pre-amplifier)

The distance between the transducers and the samples is 20 cm. The duration of the signal plays an important role; its spectrum must verify the condition of high frequency approximation referred to in the previous section. The parameters of the first investigated plastic foam M are: resistivity σ = 2500 $Nm^{-4}s$, thickness 4.1 cm, The value of the porosity given by the porosimeter [22] is ϕ = 0.97±0.02, and the value of the tortuosity α_∞, viscous and thermal characteristic length Λ, Λ' given by classical method [7-9, 19, 21] is $\alpha_\infty = 1.06 \pm 0.08$, Λ = 230 µm and Λ' = 460 µm.

Figures 3(a), 3(b), 3(c) and 3(d) show the incident signal generated by the transducer (dashed line) and the reflected signal by the plastic foam M (solid line) and their spectra for different incidents angles ($\theta = 0^0$, 17°, 23^0 and 35^0). From the spectra of these signals, the reader can see that they have practically the same bandwidth which means that there is no dispersion. The reflected signal from the foam M is very small compared with the incident signal because of the low value of its flow resistivity

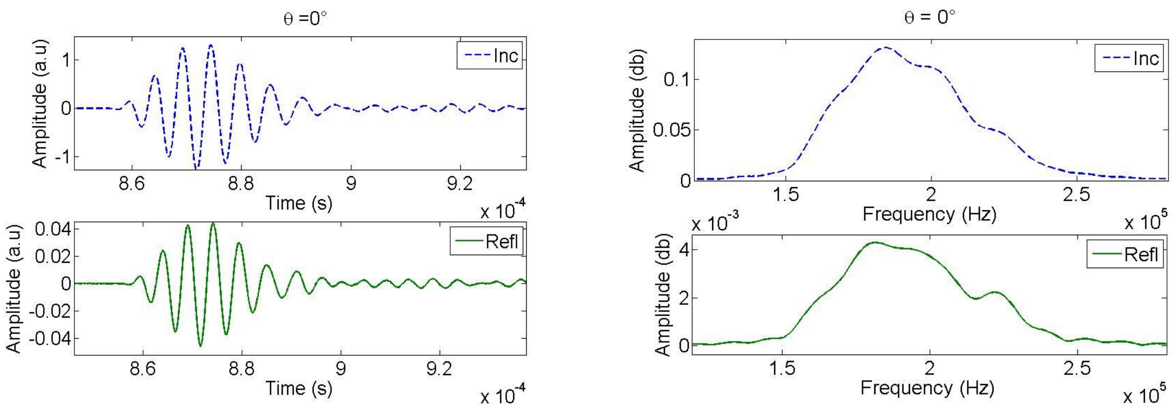


Fig.3(a)- Experimental incident signal (dashed line) and experimental reflected signal (solid line) at left and their spectrum at right (θ =0°).

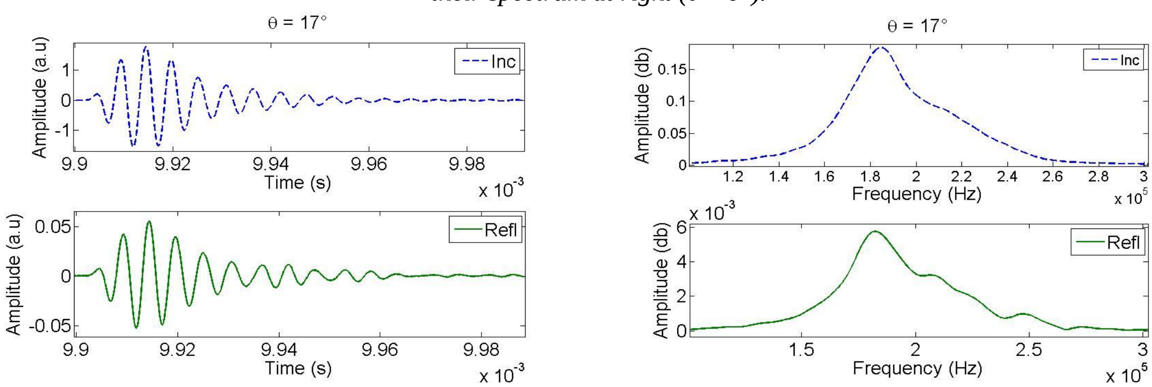


Fig.3(b)- Experimental incident signal (dashed line) and experimental reflected signal (solid line) at left and their spectrum at right (θ =17°).

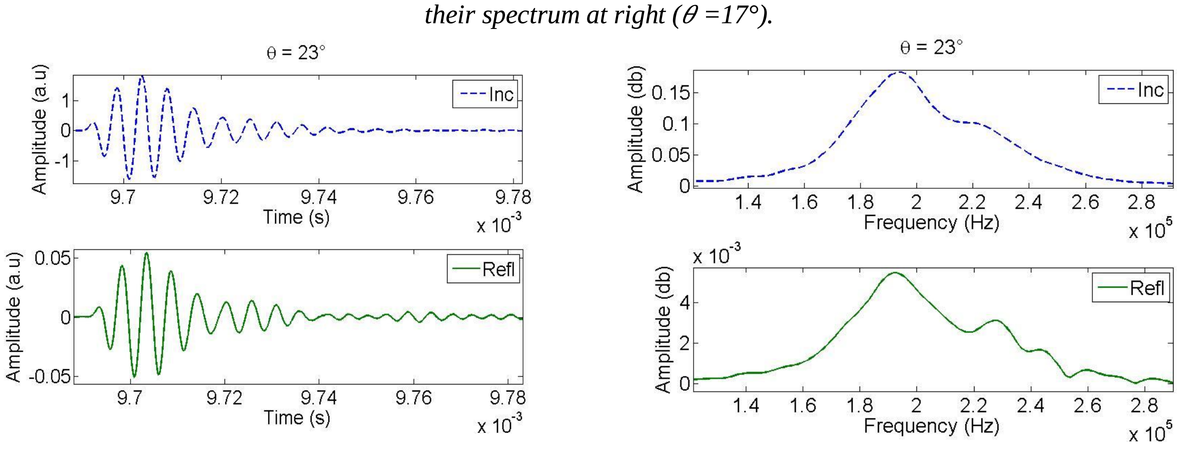


Fig.3(c)- Experimental incident signal (dashed line) and experimental reflected signal (solid line) at left and their spectrum at right (θ =23°).

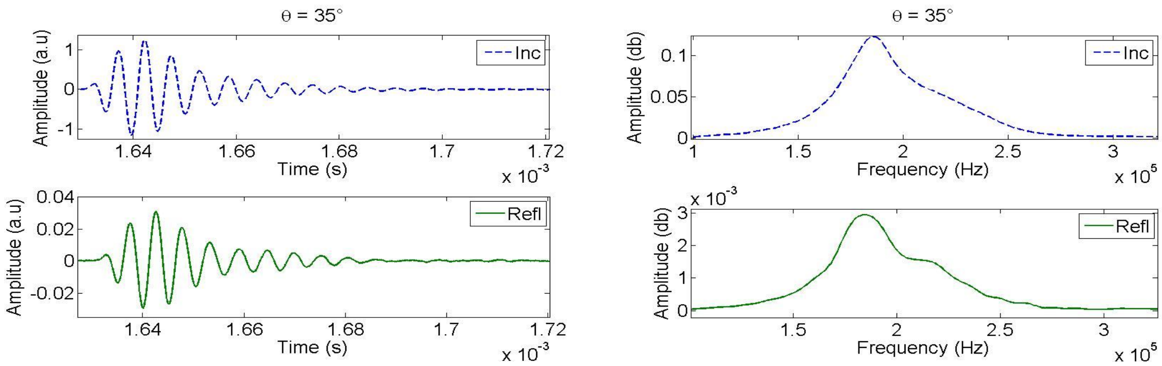
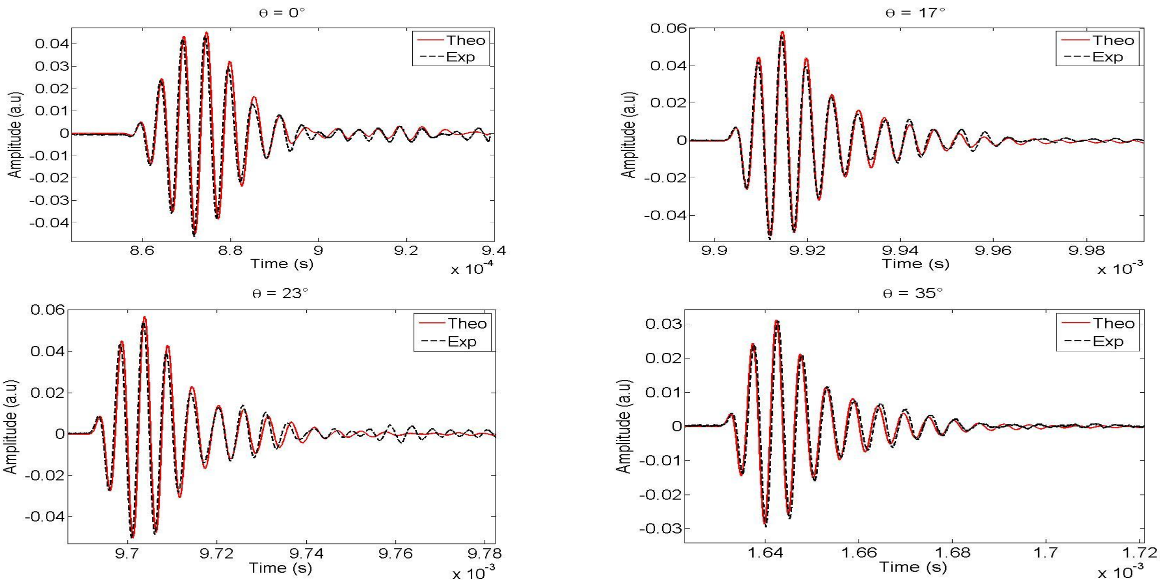


Fig.3(d). Experimental incident signal (dashed line) and experimental reflected signal (solid line) at left and their spectrum at right (θ =35°).

Figures. 4 show the comparison between experimental reflected signal (dashed line) and simulated signal (solid line) given by (18) for different incidents angles. In each case the correlation of theoretical prediction and experimental data is good.



Figs.4 Comparison between experimental reflected signal (dashed line) and simulated reflected signal (solid line) for the plastic foam sample M (θ_0 =0°, 17°, 23° and 35°)

5 Conclusion

In this paper, analytical expression of reflected coefficient at oblique incidence is calculated at frequency domain. The reflected field is obtained by convolution between the reflected operator and the incident field. Experimental validation using reflected waves at oblique incidences by air-saturated porous medium with rigid frames was performed at high frequency and found to produce excellent agreement between theory and experiment. This leads to the conclusion that the expression of reflected coefficient obtained is correct. One future hope is to solve the inverse problem and return to the physical parameters; porosity, tortuosity, viscous and thermal characteristic length of the medium, from reflected experimental data.

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